S. S. College. Jehanabad (Magadh University)

Department : Physics

Subject : Thermodynamics

Class : B.Sc(H) Physics Part I

Topic: Application of Maxwell's Thermodynamical Relation

Teacher : M. K. Singh

- Using Maxwell's thermodynamical relations, to prove that the ratio of the adiabatic to the isobaric coefficient of volume expansion is

$$\frac{1}{(1-\gamma)}$$

Adiabatic coefficient of volume expansion is given by

$$\alpha_S = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_S \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Isobaric coefficient of volume expansion is given by

$$\alpha_P = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Dividing the above two equations

$$\frac{\alpha_S}{\alpha_P} = \frac{\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_S}{\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Bringing the first term in the numerator to the denominator

$$= \frac{1}{\left(\frac{\partial T}{\partial V}\right)_S \left(\frac{\partial V}{\partial T}\right)_P}$$

Using Maxwell's Relation

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

Substituting in the above equation

$$\frac{\alpha_S}{\alpha_P} = -\frac{1}{\left(\frac{\partial P}{\partial S}\right)_V \left(\frac{\partial V}{\partial T}\right)_P}$$

Splitting the first term of the denominator

$$= \frac{1}{-\left[\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial T}{\partial S}\right)_V \left(\frac{\partial V}{\partial T}\right)_P\right]}$$

Taking the second term of the denominator to the numerator and multiplying by T

$$= \frac{T\left(\frac{\partial S}{\partial T}\right)_V}{-T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P}$$

We know that

$$C_P - C_V = T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P$$

and

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V$$

Substituting these values in the above equation we get

$$\frac{\alpha_S}{\alpha_P} = \frac{C_V}{-(C_P - C_V)}$$

$$\frac{\alpha_S}{\alpha_P} = \frac{1}{1-\left(\frac{C_P}{C_V}\right)}$$

Dividing the numerator and denominator by C_V

$$\frac{\alpha_S}{\alpha_P} = \frac{1}{1-\gamma} \qquad \text{................ (4)}$$